Is a Financial Market Bubble Closely Related to a Real Asset Bubble?

Ky-hyang Yuhn
Department of Economics
College of Business
Florida Atlantic University
Boca Raton, FL 33431
E-mail: yuhn@fau.edu
Tel: (561) 297-3224
(Corresponding Author)

and

Sang Bong Kim
Department of Economics
Hansung University
Seoul, Korea
E-mail: brainkim75@hansung.ac.kr
Tel: 82-2-760-8038

and

Joo Ha Nam
Department of Economics
Sogang University
Seoul, Korea
E-mail: namjh@ccs.sogang.ac.kr
Tel: 82-2-705-8698

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Abstract

Existing studies on bubbles have been mainly concerned with investigating the stationarity properties of stock prices and dividends. However, the standard unit-root and cointegration tests may not be able to detect an important class of bubbles. We develop a model that relates bubble measures to the Weibull distribution. In recent times there were several eruptions and subsequent collapses of seeming bubbles: 1987, 2000, and 2007. Using U.S. monthly data from 1980:1 to 2007:10, we have found that only the boom and crash of the stock market in 2007 represented an explosive bubble, although our stationarity tests fail to detect the bubble. Our empirical evidence suggests that an explosive bubble in stock prices is closely related to a housing bubble.

Key Words: Stationarity, unit roots, cointegration, duration dependence, Weibull distribution, bursting rate

JEL Classification Number: C16, C41, G12

1. Introduction

The U.S. stock market has been increasingly volatile in recent years. During one infamous week in October, 2008 (October 6 – 10), the Dow Jones lost more than 18 percent of its value, and the Dow's swing from high to low on October 10, 2008 was the biggest since the Dow Jones Index was created in 1896. There were several eruptions and subsequent collapses of seeming bubbles in the U.S. stock market in modern times. The U.S. stock market reached a peak in October 1987, which was followed by a sudden downturn commonly known as the Black Monday. The U.S. stock market reached another plateau in August 2000, but it plunged together with the burst of the IT bubble. The third peak point on the stock price series occurred in October 2007, but the stock market began to go into a tailspin, as the subprime mortgage crisis was on the horizon.

The drop of stock prices during the 1987 Black Monday was a temporary adjustment of the market. The boom of the stock market starting in the middle of the 1990s and lasting until August 2000 reflected buoyant productivity growth brought about by the IT (information technology) revolution in the latter half of the 1990s. However, the recent stock market crash distinguishes itself from the previous ones in that it occurred concomitantly with the boom and the subsequent bust of the housing market. The stock market crash that triggered the Great Depression also coincided with the construction boom and bust in 1929. Many studies have reported that the U.S. stock market before the outbreak of the Great Depression contained a speculative bubble. Thus the 2007-08 stock market crash seems to bear the stamp of the 1929 stock market crash in that the two crashes had something to do with the explosion of real estate bubbles.

Gallegati, Greenwald, Richiardi, and Stiglitz (2008) and Reinhart and Rogoff (2009) have also noted that there is a close link between financial and real asset bubbles. In particular, Gallegati, et. al. observe that "interdependencies in real and financial assets are beneficial from a social point of view when the economic environment is favorable and detrimental when the economic environment deteriorates." This paper is primarily concerned with whether the recent stock market boom and crash contained an explosive bubble, exhibiting different characteristics from the 1987 and 2000 market disturbances.

If no bubble were present in stock prices, then there would be a gradual adjustment of the stock market, and it would not take much time for stock prices to return to their fundamental values even though stock prices seem to have risen too much. However, if stock prices contained an explosive bubble, the adjustment process would be much longer and costlier, because the increased volatility of the outcome reduces the ability of the financial market to provide risk-sharing.

Earlier views on a bubble ascribe it to some psychological factors such as herd behavior, animal spirits or cognitive biases in which bubbles propagate themselves. A significant deviation of stock prices from the path predicted by market fundamentals may be due to waves of pessimistic or optimistic market psychology. Since this view attempts to link a bubble to some extraneous factors, this type of a bubble may be termed an extraneous bubble. It has been widely believed that a bubble, especially an extraneous bubble does not lend easily itself to direct testing. Bubbles are recognized only when they burst. Evans (1991) maintains that it is difficult to test for the presence of a bubble when stock prices are too high but do not have the possibility of bursting. Flood and Hodrick (1990) expressed a more pessimistic view:

> "Whether the actual volatility of equity returns is due to time variation in the rational equity risk premium or to bubbles, fads, and market inefficiencies is an open issue. Bubble tests require a well-specified model of equilibrium expected returns that has yet to be developed, and this makes inference about bubbles quite tenuous."

For reasons of this difficulty, economists traditionally used indirect methods to test for bubbles. For instance, Shiller (1981) used variance bounds tests and interpreted excessive deviations from the bounds as a bubble. However, recent bubble models elaborated by Hamilton and Whiteman (1985), Diba and Grossman (1988a,b,c), Evans (1991), Froot and Obstfeld (1991) and others have formulated rational bubbles in terms of market fundamentals, thus representing a significant departure from the traditional view. These bubbles may be termed intrinsic bubbles, as opposed to extraneous bubbles. One attractive feature of the intrinsic bubble specification may be found in its ability to derive testable implications for bubbles.

A number of recent studies have investigated the existence of intrinsic bubbles in stock markets by examining the stationarity properties of stock prices and dividends on the basis of unit-root and cointegration tests. These studies include Campbell and Shiller (1987, 1988), Diba and Grossman (1988), Crowder and Wohar (1988), Froot and Obstfeld (1991), Craine (1993), Charemza and Deadman (1995), Timmermann (1995), Lamont (1998), Bohl (2003), Sarno and Taylor (2003), Nasseh and Strauss (2004), Koustas and Serletis (2005), Cunado, Gil-Alana, and de Gracia (2005, 2007), among others.

However, the existing bubble models still remain unsatisfactory. The empirical relevance of these intrinsic bubble models has been seriously undermined by the results of Evans (1991) who has criticized that unit-root approaches are unable to detect an important class of rational bubbles. Evans has shown that even in the presence of bubbles, stock prices could be cointegrated with market fundamentals, so that standard unit-root and cointegration tests are not appropriate for detecting this class of rational bubbles. This view has been echoed by Charemza and Deadman (1995) and Ackert and Smith (1993). Ackert and Smith have noted that conventional measures of dividend payments to shareholders grossly underestimate the total cash flow to shareholders, and this underestimation of cash flows could have impacted conventional types of cointegration between dividends and prices, which may bias the tests to show no cointegration.

More interestingly, Rappoport and White (1991) have pursued a different strategy for testing for intrinsic bubbles from standard tests by directly extracting an estimate of the path of the bubble and its probability of bursting. Abreu and Brunnermeier (2003) also provide a setting in which new events can have a disproportionate impact relative to their intrinsic informational content without reference to the stationarity processes of stock prices and market fundamentals.

Another line of research on testing for speculative bubbles is provided by McQueen and Thorley (1994), Chan, McQueen, and Thorley (1998), and Harman and Zuehlke (2004) who have used hazard models to investigate duration dependence in stock prices. In particular, McQueen and Thorley have proposed that negative duration dependence in runs of positive abnormal returns is indicative of rational speculative bubbles. These studies pronouncedly differ from stationarity tests by examining the question of whether stock prices tending toward cycle lengths can be framed as one of duration dependence. However, Harman and Zuehlke have been skeptical about the validity of the duration dependence tests. They are indirect tests in nature and thus they have some limitations in testing for the presence of speculative bubbles.

The main purpose of this study is to develop a simple method of testing for rational bubbles that does not rely on the stationarity properties of variables in asset pricing models, but directly relates explicit bubble measures to the probability of bursting in the context of the Weibull distribution. To this end, we directly derive a bubble measure from the information error model and test whether the bubble measure is explosive. Our model bears a parallel to that of Rappoport and White in that we explicitly extract a bubble measure and to that of Abreu and Brunnermeier in that bubbles can arise as a result of informational errors. One novel feature of our model is that unlike the duration dependence tests conducted by McQueen and Thorley (1994) and Harman and Zuehlke (2004), we relate the bubble measures directly to the Weibull distribution. In this study we have shown that if the hazard function is estimated to be a weakly convex increasing function, the bubble is explosive, and vice versa.

We have tested for the existence of explosive bubbles in the U.S. stock market using monthly data from January 1980 to October 2007. This study notes that a test for the presence of a bubble on the basis of a long extended period of time could dilute away bubble components and thus may fail to detect it. This point is evidently made by Rappoport and White (1991). In this paper, the entire sample is segmented into several briefer periods of seeming eruptions and collapses of stock prices. Thus, we have divided the sample into three sub-samples: 1980:1 - 1987:10, 1987:11 - 2000:8, and 2000:9 - 2007:10. Our empirical results reveal that only the surge of stock prices peaking in October 2007 contained an explosive bubble. Thus, our empirical evidence shows that an explosive bubble in the stock market seems to be closely related to explosive increases in housing prices.

The paper is organized as follows. Chapter 2 briefly reviews the literature. In chapter 3, we develop a model that can be used to extract bubble measures. Chapter 4 presents empirical results and analyses. A summary and concluding remarks are provided in chapter 5.

2. A Review of the Literature

Shiller (1981), using the present value (PV) model of stock prices, has argued that if the variance of the market price of a stock is greater than that of the present discounted value of future cash flows, then stock prices are too volatile to be consistent with the present value of rationally expected future dividends discounted by a constant real interest rate. Thus, the violation of the variance bounds is interpreted as rejection of the efficient markets hypothesis.

Hamilton and Whiteman (1985) have dealt with bubbles that can be dependent on market fundamental values for the first time. They have viewed a bubble as the output of rational reactions to market fundamentals by market participants and discussed bubbles from the perspective of the stationarity properties of stock prices and market fundamentals. Their proposition implies that there exists a bubble if d-th differenced fundamental values are stationary, but d-th differenced stock prices are nonstationary. Diba and Grossman (1988) postulate that if there is no bubble premium in stock prices, then stock prices should be cointegrated with market fundamentals in a nonlinear fashion. Diba and Grossman have conducted cointegration tests for the U.S. data of the S&P 500 composite price index for the period 1871-1986 and found that the U.S. stock prices did not contain explosive rational bubbles.

Campbell and Shiller (1987) have tested for cointegration between stock prices and dividends using annual data for the S&P 500 index from 1871 to 1986, and found persistent deviations of stock prices from the present value model, which can be interpreted as evidence for the presence of rational bubbles in U.S. stock prices. In their subsequent paper, Campbell and Shiller (1989) have suggested that the dividend-price ratio (D/P) can be explained by some market fundamentals. They have tested for the model using S&P data for 1871-1986 and 1926-1986 and found that the log dividend-price ratio has a significant relationship with the growth of dividends. Their results further indicate that there is also substantial unexplained variation in the log dividend-price ratio.

Yuhn (1997) has argued that Campbell and Shiller’s (1987) linear cointegrating relation between stock prices and dividends is not appropriate for investigating stock price volatility and derived a dynamic form of cointegration between stock prices and dividends. His empirical results reveal that little evidence for linear cointegration is found, but the evidence of nonlinear cointegration is overwhelming for U.S. monthly data from 1959:1 to 1992:6, indicating no volatility in the U.S. stock market during the sample period.

Evans (1991) has conducted the simulation of two hundred replications with each of 100 ‘years’ of stock price and dividend data and shown that the cointegration of stock prices with dividends cannot be viewed as evidence against the presence of bubbles. His Dickey-Fuller unit-root tests indicate that stock prices (P) and dividends (d) are clearly cointegrated, but their simulations show that over the sample, there appear to be four bubble eruptions, each followed by a collapse.

Froot and Obstfeld (1991) have maintained in line with Hamilton and Whiteman that since a bubble is the output of market participants' rational reaction to market fundamentals, bubbles are not extraneous. They tested for a unit root in the price-dividend ratio using the S&P 500 index for the period 1900-1988, and were unable to reject the unit root null hypothesis in five of six cases. Thus, they have found evidence that is consistent with the presence of rational bubbles in stock prices.

Craine (1993) has argued that a unit root in the price-dividend ratio (P/D) violates the no rational bubbles restrictions. Using annual S&P 500 data from 1876 to 1988, he has found that either the price-dividend ratio contains a rational bubble or the discount factor must be stochastic and contain a large predictable component. Koustas and Serletis (2005) have investigated the behavior of the dividend-price ratio (D/P) or the dividend yield to test for the existence of bubbles. They have maintained that the presence of a unit root in the log dividend yield is consistent with rational bubbles in stock prices. They have applied fractional cointegration techniques to the S&P 500 log dividend yield for the period 1871 to 2000 and found that the log dividend yield is mean reverting.

Cunado et.al. (2005) have explored whether the NASDAQ composite index and its corresponding dividend yield (D/P) satisfy the condition required for the absence of rational bubbles, employing fractional cointegration methods for monthly, weekly, and daily data over the period 1994:06-2003:11. They were not able to reject the unit root null hypothesis for monthly data, but they found the order of integration to be smaller than 1 for daily and weekly data, thus, rejecting the existence of rational bubbles. Cunado et. al. (2007) have tested for the existence of bubbles in the S&P 500 index using the price-dividend ratio for the period 1871:1-2004:6. They have found orders of integration for the log price-dividend ratio to be equal to or higher than 1 and concluded that there exists a stock market bubble in the S&P 500 index over the entire period.

Bohl (2003) has studied the presence of a bubble in annual (1871-1999) and monthly (1871-2001) U.S. stock prices using the momentum threshold autoregressive (MTAR) model developed by Enders and Granger (1998). His findings indicate the absence of periodically collapsing bubbles in the U.S. stock market over the 1871-1995 period. However, the evidence for the sample including the rapid share price increases since the middle of the 1990s (1871-2001) is interpretable in favor of the existence of periodically collapsing bubbles in U.S. stock prices. Nasseh and Strauss (2004) have applied panel cointegration testing and estimation methods to quarterly data for 84 firms over the 1979-1999 period and examined the long-run relation between stock prices and dividends. Their results show that there is an approximately one-for-one long run relation (cointegrating relation) between stock prices and dividends for large established firms. However, their test results show that stock prices are overvalued by 43% during the late 1990s.

Sarno and Taylor (2003) provide some international evidence. They have examined the existence of rational bubbles in Latin American emerging markets—Brazil, Chile, Columbia, Mexico, and Venezuela. They fail to reject the null hypothesis of no cointegration, providing strong evidence for the existence of bubbles in each of the Latin American stock markets.

Unlike the existing studies that are primarily concerned with the stationarity properties of stock prices and dividends, McQueen and Thorley (1994), Cochran and Defina (1995), and Harman and Zuehlke (2004) on one the hand and Rappoport and White (1991) and Abreu and Brunnermeier (2003) on the other hand have focused on the nature of bubble measures. First, McQueen and Thorley have adopted duration dependence tests to investigate the presence of speculative rational bubbles. They have argued that if bubbles are present, then the probability that a run (sequence of observations of the same sign) of positive abnormal returns ends declines with the length of the run (positive duration dependence or negative hazard function). The estimates reported by McQueen and Thorley are consistent with the presence of speculative bubbles in the New York Stock Exchange (NYSE). In contrast, Chan, McQueen, and Thorley (1998) have applied the same discrete hazard model to weekly returns on the S & P 500 Index and found no evidence of duration dependence.

However, MQueen-Thorley’s duration dependence tests have been challenged by Harman and

Zuehlke who question the efficacy of using measures of duration dependence to test for speculative bubbles. Using both equal-weighted and value-weighted portfolios of all NYSE stocks from 1927 through 1997 and equal-weighted and value-weighted NYSE-AMEX indices for the period 1963 through 1997, they have found that evidence of duration dependence is sensitive to the choice of sample period, the method of correcting for discrete observations of continuous duration, the use of value-weighted versus equally weighted portfolios, and the use of monthly versus weekly runs of abnormal returns.

Rappoport and White have argued that although standard tests find no bubbles in the stock price data for the last 100 years, historical accounts, focusing on briefer periods, point to the existence of a bubble during the Great Depression period of 1928-29. Their approach has used the behavior of the premia demanded on loans collateralized by the purchase of stocks as a bubble measure. Abreu and Brunnermeier (2003) have developed a dynamic game model in which bubbles can persist even though all rational arbitrageurs know that the stock price is too high, and they jointly have the ability to correct the mispricing. There can be a large and long-lasting departure from fundamental values, because there is dispersion of opinions among rational arbitrageurs concerning the timing of the bubble. Their model provides a setting in which 'overreaction' and self-feeding price processes lead to full-fledged crashes.

3. The Theoretical Model

The present value (PV) model of stock prices implies that stock prices are equal to the present value of future cash flows such as dividends discounted by a constant real interest rate. Generally, the present value model takes the following form:

(1) $$P_t = \delta E_t (P_{t+1} + D_{t+1})$$

where P_t represents real stock prices at time *t* and D_{t+1} indicates real dividends between *t* and *t*+1, and δ is a discount factor. The discount factor is equal to 1/(1 + *r*) with a constant real interest rate of *r*. The solution to equation (1) is given by

(2) $$P_t = \sum_{k=1}^{\infty} \delta^k E_t (D_{t+k}) + \lim_{T \to \infty} E_t \delta^T P_{t+T}$$

If we impose a transversality condition on equation (2), then we obtain the unique solution to equation (1), which is given by

(3) $$P_t = \sum_{k=1}^{\infty} \delta^k E_t(D_{t+k})$$

If the transversality condition fails to hold, we have a bubble part:

(4) $$P_t = \sum_{k=1}^{\infty} \delta^k E_t(D_{t+k}) + B_t$$

where $B_t = \lim_{T\to\infty} E_t \delta^T P_{t+T}$ measures the bubble term and satisfies the following process:

(5) $$B_t = \delta E_t B_{t+1}$$

However, Hamilton (1986) and Diba and Grossman (1988a) reformulate the PV model in a different way from the conventional one such as equation (4). Hamilton (1986) has proposed the following PV model:

(6) $$P_t = \delta(D_{t+1} + E_t P_{t+1} + \pi_t)$$

where π_t can be viewed as a catch-all random variable that is not observed by the researcher, but can be observed only by market participants. The random variable includes the real interest rate, the risk premium, taxes on dividends, etc. π_t is assumed to be stationary, that is, I(0).

Diba and Grossman (1988a) have conducted an empirical investigation of the Hamilton model using a slightly modified version.

(7) $$P_t = \delta E_t(P_{t+1} + \alpha D_{t+1} + \pi_{t+1})$$

Diba and Grossman have proposed that if the unobserved π_t is stationary, and first differenced dividends and first-differenced stock prices are stationary, no bubble is present. That is, there is no bubble if π_t is I(0), and stock prices and dividends are I(1), and stock prices and dividends are cointegrated (CI(1,1)). Diba and Grossman have derived the following estimation equation:

(8) $$P_{t+1} + \gamma D_{t+1} - (1/\delta)P_t = e_{t+1} - \pi_{t+1}$$

where $e_{t+1} = P_{t+1} + \gamma D_{t+1} + \pi_{t+1} - E_t(P_{t+1} + \gamma D_{t+1} + \pi_{t+1})$ .The left-hand side becomes stationary if π_{t+1} is stationary, since e_{t+1} is not serially correlated.

In this study, we develop a rational bubble model in the spirit of Hamilton (1986) and Diba and Grossman (1988) but in a more straightforward manner. Our bubble model can be viewed as a significant improvement over the Hamilton and Diba-Grossman approaches. The obvious merit of our approach is that we derive the bubble measure without introducing unobservable random variables. Our model formulates a bubble measure as an overreaction to new information on market fundamentals by rational participants.

Since the present value relation must hold in period t - 1, we have

(9) $$P_{t-1} = \sum_{k=1}^{\infty} \delta^k E_{t-1}(D_{t+k-1})$$

Multiplying equation (3) by δ and subtracting it from equation (9),

(10) $$P_{t-1} - \delta P_t = E_{t-1}\delta D_t - \sum_{k=1}^{\infty} \delta^{k+1}[E_t - E_{t-1}]D_{t+k}$$

We can rearrange equation (10) to obtain

(11a) $$\delta(P_t + D_t) - P_{t-1} = \sum_{k=0}^{\infty} \delta^{k+1}[E_t - E_{t-1}]D_{t+k}$$

(11b) $$P_t + D_t = \frac{1}{\delta}P_{t-1} + \sum_{k=0}^{\infty} \delta^k [E_t - E_{t-1}]D_{t+k}$$

Equation (11b) can be further simplified as

(12) $$P_t = (1+r)P_{t-1} - D_t + \nabla_t$$

where $\nabla_t = \sum_{k=0}^{\infty} \delta^k \, [E_t - E_{t-1}] \, D_{t+k}$ denotes the present value of the sum of the forecast errors of market participants. That is, this is the rational response of participants to market fundamentals since $[E_t - E_{t-1}]D_{t+k}$ is the difference between the k-period-ahead forecast of dividends in the previous period and the k-period-ahead forecast of dividends based on the arrival of new information in the current period. Since the bubble measure can be viewed as a reaction to new information on market fundamentals or an information update by market participants, equation (12) will be called an information error model.

It can be demonstrated that the term ∇_t is essentially the same as the unobserved variable $(e_t + \pi_t)$

in the Hamilton-Whiteman and Diba-Grossman models. We can rewrite our information error model as

(13) $$P_{t+1} = \frac{1}{\delta} P_t - E_t D_{t+1} + v_{t+1}$$

where $v_{t+1} = \sum_{k=1}^{\infty} \delta^k [E_t - E_{t-1}] D_{t+k+1}$. Let $E_{t+1} D_{t+1} - E_t D_{t+1} = e_{t+1}$

(14) $$P_{t+1} + D_{t+1} - \frac{1}{\delta} P_t = e_{t+1} + v_{t+1}$$

Equation (14) is the same as the Diba-Grossman model when γ = 1 in their model. We note that $\nabla_{t+1} = e_{t+1} + v_{t+1}$. ($\nabla_{t+1} = e_{t+1} + \pi_{t+1}$ in the Diba-Grossman model). Thus, we can extract bubble measures from $P_t + D_t - (1/\delta) P_{t-1}$ without introducing arbitrarily an unobserved random variable. Furthermore, ∇_t is not serially uncorrelated. This can be proved by the law of iterated expectations.

It is assumed that the bubble measure (∇_t) has a Weibull distribution. There is not only a parallel between the burst of a speculative bubble and a material's burning out, but also there is a good reason to believe that the bubble measure can be appropriately modeled as the Weibull function. A bubble is a rare event. Like other rare events, bubbles can be formulated in terms of the instantaneous rate at which an event occurs after duration t since some prior event has occurred.

We will denote a bursting bubble by a continuous random variable *T*. If *T* has a probability density function *f*(*t*), then the probability that optimistic expectations about stock prices continue to hold until a specific time *t* is given by

(15) $$z(t) = \Pr(T > t) = \int_t^{\infty} f(t)dt = 1 - F(t)$$

where *z*(*t*) is the survival function for optimistic forecasting until a specific time *t*, and *F*(*t*) is the cumulative density function. Then the following rate measures the likelihood of the bursting of a bubble in the next small unit of time Δ*t*, given that a bubble has survived until time t.

(16) $$\theta(t) = \lim_{\Delta t \to 0} \frac{[F(t+\Delta t) - F(t)]}{\Delta t} \cdot \frac{1}{z(t)} = \frac{F'(t)}{z(t)} = \frac{f(t)}{z(t)} = \frac{f(t)}{1 - F(t)}$$

Thus, the busting rate is given by

(17) $$\theta(t) = -\frac{z'(t)}{z(t)} = -\frac{d \ln z(t)}{dt}$$

From equation (17), we can derive the probability density function (pdf) as follows:

(18a) $$\ln z(t) = -\int \theta(t)dt + \ln c$$

(18b) $$z(t) = ce^{-\int \theta(t)dt}$$

Then the cumulative density function can be expressed as

(19) $$F(t) = 1 - ce^{-\int \theta(t)dt}$$

If $f(t)$ follows the Weibull distribution, then we have the following probability density function

(20) $$f(t) = \alpha\lambda t^{\alpha-1} \exp(-\lambda t^{\alpha})$$

and the bursting rate is given by

(21) $$\theta(t) = \alpha\lambda t^{\alpha-1}$$

When λ is equal to 1, we have $\theta(t) = \alpha t^{\alpha-1}$, which gives the possibility of an extraneous bubble (speculative bubble). $\theta(t)$ can be greater than one since $\theta(t)\cdot\Delta t$ is equal to the conditional probability. The coefficient α denotes the shape parameter, also known as the Weibull slope. Different values of the shape parameter can have significant effects on the behavior of the distribution. In fact, different values of the shape parameter may lead to different distributions. For example, when $\alpha = 1$, the pdf of the two-parameter Weibull reduces to that of the one-parameter exponential distribution. The bursting rate $\theta(t)$ will increase or decrease depending on the value of α. There are five possibilities.

(1) If α is smaller than one, the bursting rate decreases, so that the possibility of bursting will also decrease as time goes on.

(2) If α is equal to one, the bursting rate is constant, so that the possibility of bursting will also be constant. This case is the same as the well-known exponential distribution.

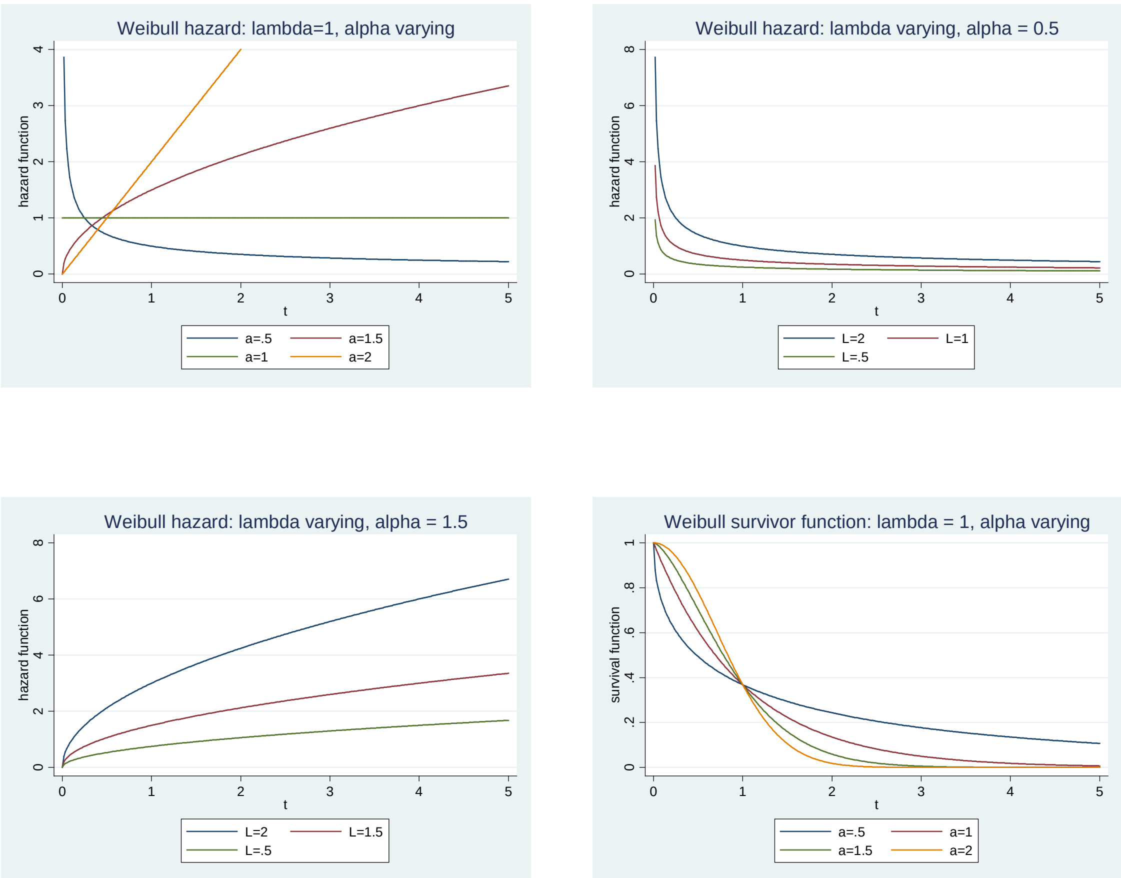
(3) If $1 < \alpha < 2$, the bursting rate increases at a decreasing rate, so that the possibility of bursting

also will increase at a decreasing rate.

(4) If α is equal to 2, the bursting rate increases at a constant rate.

(5) If α is greater than 2, the bursting rate increases at an increasing rate, so that the possibility of bursting will increase at an increasing rate. Thus, an explosive bubble occurs when the value of α is equal to or greater than 2. <Figure 1> shows the Weibull distribution for various values of the parameters.

<Figure 1> Weibull Distributions



We can derive the specifications for purely extraneous bubbles and intrinsic bubbles from equation (21). We obtain an intrinsic bubble model if we let $\lambda = \exp(X'\beta)$ and an extraneous bubble model if we let $\lambda = 1$:

(22a) $$\theta(t) = \alpha t^{\alpha-1}\exp(X'\beta) = \psi_1(t)\psi_2(X'):$$ Intrinsic bubbles

(22b) $$\theta(t) = \alpha t^{\alpha-1} = \psi_1(t) \text{ with } \psi_2(X') = 1:$$ Extraneous bubbles where X denotes the set of variables that are thought to be related to a firm's fundamental values.

We have included a set of market fundamental variables in the intrinsic bubble model that affect stock prices. Fama and French (1989), McQueen and Thorley (1994), Harman and Zuehlke (2004) and others have maintained that the term spread and the dividend yield (D/P) or the P/E ratio are useful in predicting real abnormal returns. Reinhart and Rogoff (2009) and others have argued that rising indebtedness and current-account deficits are important factors that could lead to financial crashes. In the tradition of these studies, the variables that represent market fundamentals include the term spread (TERM), the price-earnings ratio (PER), the unemployment rate (UNEMP), the default rate (DEF) and the exchange rate (EX). Thus, the intrinsic bubble specification is given by

(23) $$\theta(t) = \alpha t^{\alpha-1} \exp\left(\beta_0 + \beta_1 PER_t + \beta_2 TERM_t + \beta_3 UNEMP_t + \beta_4 DEF_t + \beta_5 EX_t\right)$$

4. Empirical Analysis

4.1. Data Description

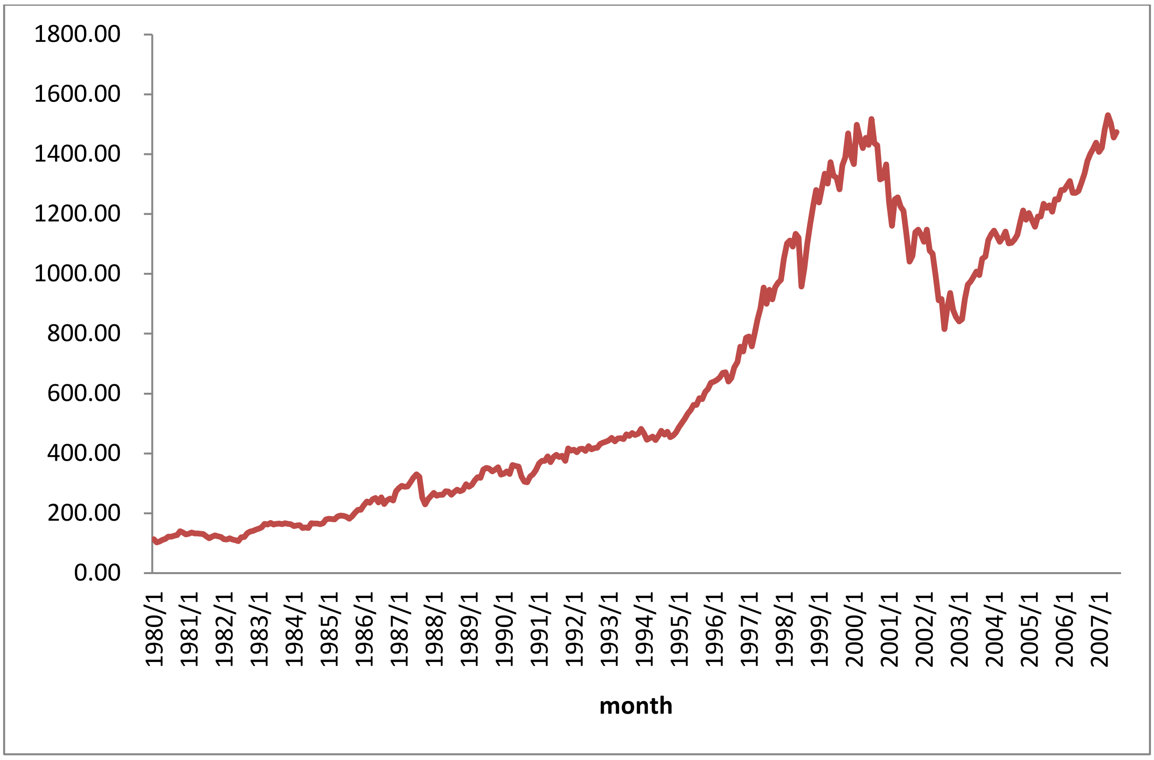
We have used U.S. monthly data from 1980:1 to 2007:10. We have divided the sample into three sub-samples: 1980:1-1987:10 (period 1), 1987:11-2000:8 (period 2), and 2000:9- 2007:10 (period 3). This breakdown of the sample coincides with the eruptions and subsequent collapses of seeming bubbles in the U.S. stock market in modern times. The stock price index reached a peak in October 1987 followed by a sudden decline in U.S. stock prices, and it reached another peak in August 2000 followed by the collapse of the stock prices together with the burst of the IT bubble. The third peak point on the series occurred in October 2007.

The data used in this study include the S&P 500 composite index, the price-earnings ratio (PER), the term spread (TERM) between the short-term interest rate (3-month T-bill rate) and the long-term interest rate (10-year T-bond rate), the default rate (DEF), the nominal effective exchange rate (EX), and the unemployment rate (UNEMP). The data have been obtained from the Datastream database and the *International Financial Statistics* (*IFS*). The price-earnings ratio (PER) is derived by dividing the total market value of an index by the total amount of earnings. The default rate (DEF) is the default rate on all

U.S. corporate bonds.

The term spread measures the forecast of the future economy. In a well-developed market, short-term interest rates reflect policy interest rates, but long-term interest rates include market participants' expectations of the future state of the economy. Thus, the term premium is an index of market participants' forecasting of the economy. The term spread contains information on future inflation and economic growth. The following diagram shows movements in the S&P 500 index from January 1980 to December 2007.

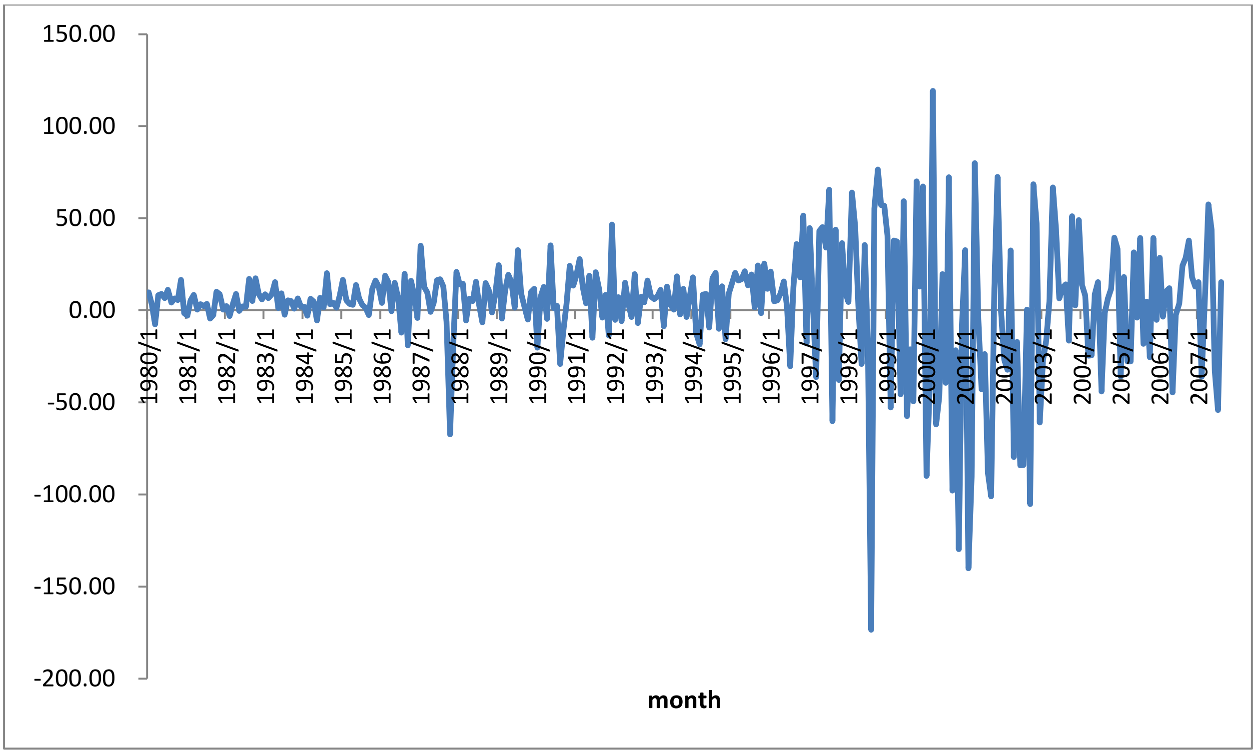
<Figure 2> S&P500 Index



4.2. Measuring Bubbles

We obtain the portion of unexplained variations of stock prices (bubble measures) from the information error model. It is not possible to judge whether stock prices are 'too high' or 'too low' without referring to market fundamentals. The information error model gives a criterion that tells us whether current stock prices are too high or too low compared to the path implied by the market fundamentals. Our information error model enables us to divide current stock prices into two parts: the market fundamental part that is given by $P_t = -D_t + \frac{1}{\delta} P_{t-1}$ and the bubble part that is measured by $\nabla_t = \sum_{k=0}^{\infty} \delta^k \, [\, E_t - E_{t-1}] \, D_{t+k}$. The bubble measure has been normalized to take positive values. The decomposition of stock prices into these two parts is available from the authors upon request.

<Figure 3> Non-Fundamentals in Stock Prices



4.3. Empirical Results

A. Tests for Unit Roots

We have first checked whether the time series data under consideration are stationary. If a variable is nonstationary, it contains a unit root. In this paper, we use the Phillips-Perron tests (PP) that allow the disturbances to be weakly dependent and heterogeneously distributed. We have conducted unit-root tests for the following two data-generating processes: (24a) has a drift term only, and (24b) has both a drift and a linear time trend.

(24a) $y_t = \mu + \alpha y^*_{t-1} + u^*_t$

(24b) $y_t = \mu + \beta(t - T/2) + \alpha y_{t-1} + u_t$

<Table 1> Unit-Root Tests

Variables	Phillips-Perron Test							
	$y_t = \mu + \alpha y_{t-1} + u_t$				$y_t = \mu + \alpha y_{t-1} + \beta(t - T/2) + u_t$			
	1980:1-2007:10	1980.1-1987.10	1987.11-2000.8	2000.9-2007.10	1980:1-2007:10	1980.1-1987.10	1987.11-2000.8	2000.9-2007.10
Bubble Measure	-18.22**	-6.30**	-15.20**	-7.86**	-18.29**	-6.33**	-15.62**	-8.69**
PER	-2.15	-1.11	-0.93	-1.53	-2.33	-2.25	-1.87	-2.34
Term Premium	-4.59**	-3.47**	-2.41	-1.43	-4.54**	-3.78**	-2.39	-2.20
Unemployment Rates	-1.58	-1.36	-0.32	-2.00	-2.81	-2.05	-1.17	-2.52
Default Rates	-2.10	-1.34	-1.46	-0.24	-1.85	-2.47	-1.39	-2.62
Exchange Rates	-2.35	-1.51	-1.14	-0.34	0.94	-0.42	-2.88	-3.02

*indicates significance at the 1% level and ** significance at the 5% level.

Our test results show that as far as the unit root tests are concerned, there is no substantial difference between the two processes.

1. The bubble measure is unambiguously stationary. The bubble measure has turned out to be stationary in both the data-generating processes for every sub-period as well as for the entire sample period.

2. The term spread is stationary only in period 1 and in the entire period.

3. We fail to reject the unit root null hypothesis for all other variables.

4. Thus, all variables except for the bubble measure in all sample periods and the term spread (in period 1 and the entire period) contain a unit root. It is necessary to make these nonstationary variables stationary before we run a regression. These nonstationary variables have been first-differenced. We have confirmed that these first-differenced variables are stationary. The test results are not reported here, but they are available from the authors on request.

B. Tests for Bubbles: The Weibull Tests

A bubble of stock prices can be detected only when there is the possibility that the bubble bursts. As long as there is no possibility of explosion even though stock prices are very high, such volatile stock prices will adjust to an appropriate level with the passage of time that is consistent with market fundamentals. As we have seen from the Weibull specification, there is the possibility of bursting if α is equal to or greater than 2.

B.1. Testing for Purely Extraneous Bubbles

Purely extraneous bubbles are associated with market participants' psychological wave of moods, not with the current or future state of the economy. Our extraneous-bubble model is represented by

(22b) $$\theta(t) = \alpha t^{\alpha-1} = \psi_1(t)$$

<Table 2> Tests for Extraneous Bubbles

Period	1980:1-2007:10	1980:1-1987:10	1987.11-2000:8	2000:9-2007:10
α	1.32*	1.37*	1.48*	1.64*

* indicates significance at the 5 percent level.

<Table 2> shows that there is no evidence for extraneous bubbles in the U.S. stock market, indicating that there is no possibility that seemingly too high stock prices become explosive bubbles driven by a wave of people's psychological biases or fads. The estimated value of α is 1.32 for the entire sample period, and 1.37 for sample period 1, 1.48 for sample period 2, and 1.64 for sample period 3. All the estimates are significant at the 5 percent level of significance. Thus, we can conclude that although stock prices seem to have been overvalued on certain time intervals, such overvaluation of stock prices was not driven by extraneous factors.

B. 2. Testing for Intrinsic bubbles

In order to investigate whether bubbles are driven by market fundamentals or not, we have estimated the following model with nonstationary variables first-differenced:

(23) $$\theta(t) = \alpha t^{\alpha-1} \exp\left(\beta_0 + \beta_1 PER_t + \beta_2 TERM_t + \beta_3 UNEMP_t + \beta_4 DEF_t + \beta_5 EX_t\right)$$

<Table 3> Tests for Intrinsic Bubbles

	With the results of the unit root test			
Period	1980:1-2007:10	1980:1-1987:10	1987:11-2000:8	2000:9-2007:10
α	1.52*	1.78*	1.61*	1.92*
β_0	0.80	-0.39	1.09	-9.14*
PER	0.06*	-0.05	0.01	-0.13*
TERM	0.04	-0.05	0.34*	0.13
UNEMP	-0.36*	-0.39*	-0.49*	0.21
DEF	0.11*	-0.18	-0.01	0.34
EX	-0.02*	0.06*	-0.01	0.08

* indicates significance at the 5 percent level.

The key element in the Weibull distribution is the size of α. When the hazard function is weakly convex increasing function (α ≥ 2), then the bursting rate increases at an increasing rate, and this is interpretable in favor of the existence of an explosive bubble. Our major findings can be summarized as follows:

1. The value of α is 1.52 for the entire sample period, and the coefficient is significant. Thus, no evidence of explosive bubbles is found for the entire sample period.

2. The value of α is considerably less than 2 at the 5 percent level of significance in both the sample period 1 (1980:1-1987:10) and sample period 2 (1987:11-2000:8). Thus, during these two sub-sample periods, stock prices seem to have risen too much, but they did not develop into explosive bubbles. The spike of stock prices followed by the largest single-day drop during the 1987 Black Monday was a temporary adjustment of the market. The upward trend in U.S. stock prices starting in the middle of the 1990s and lasting until August 1980 reflected productivity growth brought about by the IT (information technology) revolution and did not represent an explosive bubble.

3. Interestingly enough, the value of α is 1.92 for the sample period 3 (2000:9-2007:10) and

significant at the conventional level of significance. It is interesting to note that the hazard function is estimated to be less concave during the sample period 3 compared to the previous two subsample periods. Since the estimate is so close to 2 that this finding can be construed as evidence for the existence of an explosive bubble during the 2000:9-2007:10 period. The recent market crash came in the wake of the subprime crisis in the housing market. During this sample period, the stock market curve tended to move in tandem with the housing market curve. In their recent work, Gallegati, Greenwald, Richiardi, and Stiglitz (2008) and Reinhart and Rogoff (2009) have also noted that financial crashes typically follow the burst of real-estate bubbles. Thus, our empirical evidence corroborates the proposition that a stock market bubble triggered by a real estate bubble could develop into an explosive one.

4. As observed by Rappoport and White (1991), it is entirely possible to detect a bubble in briefer sub-sample periods even though bubbles may not be present over a longer period of time that encompasses the shorter sub-sample periods. A detailed comparison of our findings with other studies will be presented in section D.

C. Tests for Bubbles: Unit-Root and Cointegration Tests

Following the traditional approach to testing for the existence of explosive bubbles, we have also conducted unit-root and cointegration tests for stock prices (P) and dividends (D). We have first tested for unit roots for the P and D series with a constant only and with both a constant and a trend.

<Table 4> Phillips-Perron Unit-Root Test for Prices (P) and Dividends (D)

	$y_t = \mu + \alpha y_{t-1} + u_t$				$y_t = \mu + \alpha y_{t-1} + \beta (t - T/2) + u_t$			
Period	1980:1-2007:10	1980:1-1987:10	1987:11-2000:8	2000:9-2007:10	1980:1-2007:10	1980:1-1987:10	1987:11-2000:8	2000:9-2007:10
P	0.129 (0.974)	-0.753 (0.827)	3.152 (1.000)	-1.081 (0.720)	-1.912 (0.646)	-2.262 (0.450)	-0.184 (0.993)	-2.928 (0.159)
D	3.395 (1.000)	1.179 (0.998)	-2.117 (0.238)	1.256 (0.998)	1.386 (1.000)	-3.077 (0.118)	-6.013 (0.000)	-2.205 (0.325)
ΔP	-18.716 (0.000)	-6.584 (0.000)	-14.319 (0.000)	-8.334 (0.000)	-18.732 (0.000)	-6.329 (0.000)	-16.819 (0.000)	-8.798 (0.000)
ΔD	-25.865 (0.000)	-18.682 (0.000)	-24.066 (0.000)	-12.218 (0.000)	-28.433 (0.000)	-13.910 (0.000)	-25.953 (0.000)	-14.604 (0.000)

*Numbers in parentheses denote p-values.

Our Phillips-Perron unit-root tests reveal that stock prices unambiguously contain a unit root in both the processes for all sample periods, and the results for dividends are very similar to those of stock prices only with the exception that the dividend series (with both a constant and a trend) is stationary during sample period 2. On the other hand, first-differenced stock price series and first-differenced dividend series are stationary in all data-generating processes for all samples.

Since the P and D series are nonstationary, we have tested for cointegration between stock prices and dividends using the Augmented Engle-Granger (AEG) test. Table 5 presents the cointegration test results. We are unable to reject the null hypothesis of no cointegration for the entire sample period as well as for sub-sample periods 1 and 2, but we reject the null hypothesis for sub-sample period 3. According to the unit-root and cointegration test results, we should conclude that there were eruptions of bubbles during the 1980:1-1989:1 and 1989:2-2000:8 periods, but there was no explosive bubble during the 2000:9-2007 period. Thus, we arrive at the conclusions opposite to those of the Weibull test results.

<Table 5> AEG Test for Conintegration between Stock Prices and Dividends

	$y_t = \mu + \alpha y_{t-1} + u_t$				$y_t = \mu + \alpha y_{t-1} + \beta(t - T/2) + u_t$			
Period	1980:1-2007:10	1980:1-1987:10	1987:11-2000:8	2000:9-2007:10	1980:1-2007:10	1980:1-1987:10	1987:11-2000:8	2000:9-2007:10
AEG	-0.732 (0.969)	-2.867 (0.178)	-1.732 (0.732)	-2.901 (0.167)	-1.815 (0.695)	-2.527 (0.315)	-0.528 (0.981)	-4.911 (0.001)

*Numbers in parentheses denote p-values.

D. Comparisons with Other Studies

D.1. Comparison with Unit-Root and Cointegration Test Results

Since most of existing studies on bubbles have dealt with samples up to the late 1990s, a direct comparison of our results with those studies may not be appropriate. Bohl (2003) and Nasseh and Straus (2004) have used the most recent data series, employing unit-root and cointegration tests. Bohl has found that although the results from the subsample 1871-1995 cannot be interpreted in favor of the existence of periodically collapsing bubbles in the U.S. stock market, evidence from the 1871-2001sample period indicates their presence. On the other hand, Nasseh and Strauss have found that no bubble was present for the entire sample period of 1979:3–1999:2, but since the mid-1990s, the present-value model parameters indicate a 43% overvaluation of stock prices. These two studies have discovered that explosive bubbles were present in the U.S. stock market when the sample data include recent stock price hikes up to the late 1990s or early 2000s. Their findings based on the unit root and cointegration tests are roughly in agreement with our unit-root and cointegration test results but at variance with our Weibull test results.

D. 2. Comparison with Duration Dependence Test Results

McQueen and Thorley (1994) have found statistical evidence of negative duration dependence in runs of positive abnormal monthly returns for both the equally weighted and value-weighted portfolios of all NYSE-traded stocks from 1927 to 1991, and interpreted this finding as evidence for the presence of speculative bubbles in the NYSE. On the other hand, Harman and Zuehlke (2004) have used both equally weighted and value-weighted portfolios of all NYSE stocks from 1927 through 1997 (monthly data) and equally weighted and value-weighted NYSE-AMEX indices for the period 1963 through 1997 (weekly data). They have found that with monthly data, only the Discrete Weibull and Discrete Logistic models in conjunction with value-weighted portfolios provide evidence supporting speculative bubbles, but the Continuous and Interval Weibull models consistently yield evidence of positive duration dependence for runs of both positive and negative abnormal returns. Furthermore, neither of these models provides evidence of speculative bubbles when weekly data are used.

These duration test results seem to indicate that there is evidence in favor of a bubble in a sample that spans the stock market boom before the Great Depression, but there is no evidence of a bubble in a recent briefer sample (1963-1997). Harman-Zuehlke’s shorter data set overlaps with our data set, and their evidence in the main agrees with ours. As Harman and Zuehlke demonstrate, however, duration dependence test results are sensitive to the choice of sample period, the method of correcting for discrete observations of continuous duration, the use of value-weighted versus equally weighted portfolios, and the use of monthly versus weekly runs of abnormal returns. Thus no conclusive evidence on the presence of a bubble can be drawn from duration dependence tests.

5. Concluding Remarks

The purpose of this study is to establish theoretical foundations for rational bubbles and to provide empirical evidence on the existence of rational bubbles in the U.S. stock market. Recently, there has been a resurgence of interest in bubbles in stock prices as U.S. stock prices have exhibited wide fluctuations over the past 20 years or so. The bubble models elaborated by Hamilton and Whiteman (1985), Diba and Grossman (1988a,b,c), Froot and Obstfeld (1991) and others represent a significant departure from the conventional view in that they reinterpret rational bubbles in terms of market fundamentals. The attractive feature of the intrinsic bubble approach is its ability to derive testable implications for bubbles by investigating the stationarity properties of stock prices and dividends or by parameterizing a specific bubble relationship as a function of market fundamentals.

However, the existing approach to modeling intrinsic bubbles still remains unsatisfactory. As Evans and others indicate, unit-root and cointegration tests are unable to detect an important class of rational bubbles. Alternative strategies to the traditional stationarity tests have been proposed by Rappoport and White (1991), Abreu and Brunnermeier (2003), McQueen and Thorley (1994) and Harman and Zuehlke (2004) who have not relied on unit-root and cointegration tests. These models explore the nature of bursting bubbles from the aspect of overreaction and information updates (Rapport and White, Abreu and Brunnermeier) or from the aspect of duration dependence (McQueen and Thorley, Harman and Zuehlke).

In this study we develop an information error model that allows one to derive bubble measures in a straightforward manner. This study provides a new method of testing for bubbles by specifying bubble measures in the context of the Weibull distribution. In our model, a bubble can occur as a result of overreaction of market participants to new information.

We have tested for the presence of bubbles in the U.S. stock market using monthly data from January 1980 to October 2007. We have divided the entire sample into three sub-samples: (1) 1980:1–1987:10, (2) 1987:11–2000:8, and (3) 2000:9-2007:10. The division of the sample period into the three sub-sample periods roughly coincides with the uphill surges of stock prices followed by their downturns over the past three decades or so. The first sample period corresponds to the Black Monday episode; the second period falls in with the boom and bust of the IT frenzy; and the third sample period coincides with the ups and downs of the housing market.

Our empirical analysis reveals that there is no evidence for extraneous bubbles in the U.S. stock market in any sample period. Our study further shows that there is no evidence of intrinsic bubbles for the entire sample period as well as for the 1980:1–1987:10 and 1987:11–2000:8 sub-sample periods. However, it is worth noting that the surge and the subsequent collapse of stock prices during the 2000:9-2007:10 period turned out to be an explosive intrinsic bubble. During this sample period, the stock market curve tended to move in tandem with the housing market curve. Many studies have also confirmed the presence of a speculative bubble in stock prices before the Great Depression, which was preceded by the construction boom. Our empirical results support the proposition that explosive stock market bubbles are closely related to real-estate bubbles.

Interestingly enough, our unit-root and cointegration tests have produced opposite results. The stationarity tests show that there were explosive bubbles during the 1980:1-1989:1 and 1989:2-2000:8 periods, but there was no bubble during the 2000:9-2007 period. These results do not square with those of the Weibull tests. Criticisms of unit-root and cointegration tests by Evans and others have been reinforced by this study.

Finally, a promising area for future research is to develop a model that integrates both stock market bubbles and real-estate bubbles into one framework. The authors are currently working on this research topic.

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